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NOTES ON THE TRANSMISSION OF PTOLEMY'S *ALMAGEST* AND SOME GEOMETRICAL MECHANISMS TO THE ERA OF COPERNICUS

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Abstract: We trace the transmission of Ptolemy's *Almagest* from the time of its composition to Arabic translations, Latin translations, and the *Epitome of the Almagest* by Peurbach and Regiomontanus. Along the way, Ptolemaic astronomy and Aristotelian science acquired some new features in thirteenth century Marāgha and fourteenth century Damascus which may or may not have been transmitted to Copernicus himself. While present evidence still indicates that Copernicus derived his geodynamic, heliocentric system on his own, we now have evidence for the transmission of some 'geometrical mechanisms' from Asia Minor to Italy and beyond after the fall of the Byzantine Empire in 1453. It stands to reason that if documents by medieval scholars appeared in Western Europe, and Renaissance scholars who understood these documents traveled to Italy at the time Copernicus was there, some of these geometrical mechanisms could have found their way into Copernicus' work.

Keywords: Ptolemy's *Almagest*, Toledo School of translators, Copernican theory

1 PTOLEMY FROM GREEK TO ARABIC

The most significant astronomical work handed down to us from ancient Greek times was written by Claudius Ptolemy (ca. AD 100–170). It is known as the *Almagest*, though its formal title was *Mathematike Syntaxis* (or mathematical compilation). It contains a star catalogue—probably significantly dependent on the star catalogue of Hipparchus (ca. 150 BC)—and a detailed summary of an elaborate geocentric model of the Universe, with the Earth at the center, and with the Moon, planets, Sun, and stars revolving around the Earth on a daily basis. The starry sky was considered to be *immutable*. The position of each star in its respective constellation, and the brightness of each star, was considered to be unchanging. The planets (i.e., wandering stars), the Moon, and the Sun appear to move through the constellations of the zodiac. The Moon always moves east against the background of stars, but the planets are observed to move east against the stars (direct motion) or west against the stars (retrograde motion), depending on the particular Sun-Earth-planet geometry (Toomer, 1975; 1978).

In addition to adopting a geostatic and geocentric model of the Universe, the Greeks accepted Aristotle's physics, in so far as motion in the heavens must be described by uniform circular motion, whereas linear motion is a characteristic only of the sublunar realm, i.e. the Earth, its atmosphere, and the volume of space extending to the Moon. As described simplistically in many textbooks and non-technical books on the history of astronomy, the Moon and planets revolve around the Earth, each associated with its own circular *deferent*.[1] To account for retrograde motion or a variation of distance from the Earth, the Greek model used one or more *epicycles* for each body (e.g., see Evans, 1998). When Mars, for example, is in opposition to the Sun, we observe the mid-point of its retrograde motion. The Greek model stipulates that the speed of the planet on the epicycle is faster than the speed of the center of the epicycle along the deferent. At opposition the vector sum of these two velocities produces the westward motion of the planet with respect to the stars. Even if one has determined the best epicycle and deferent for a given object, a comparison of celestial positions based on a particular model to actual measurements led to the use of more than one epicycle per object, or one could use an *eccentric* deferent (i.e., one whose center is offset from the position of the Earth). Or, one could use an *equant*, whereby the Earth is offset from the center of a planet's deferent, and the center of an epicycle moves uniformly in *degrees per unit time* with respect to a point opposite the center of the deferent from the Earth's position (see Gingerich, 1986: 80–81). To be more accurate historically, there were no 'orbits' in pre-modern astronomy. A deferent was conceived of as a solid spherical body or the equator of that body. A planet is carried on an epicycle whose center is carried by a deferent. One key point is that an ancient Greek or medieval model of the motion of a celestial body had as its primary goal to account for its past and/or future positions in the sky. The *direction* to the body was key, rather than the implied physical distance to the object.

It is not our purpose here to present an overview of the *Almagest*. For that we direct the reader to Pedersen's (1974) monograph, *A Survey of the Almagest*, and to Toomer's (1984) masterful translation, which contains many useful added figures. We have a much more modest goal: to briefly trace the transmission of the *Almagest* up to the time of Copernicus.

The *Almagest* was written in Greek and finished around 150 AD. According to Sarton (1975), the first Arabic translation was made in the early years of the ninth century by Sahl al-Tabarī, a Jewish scholar from Tabaristan (northern modern-day Iran, bordering the Caspian Sea). More significantly, under the auspices of the Abassid Caliph Ma'mūn (who ruled Baghdad from 813 to 833), al-Hajjāj ibn Yūsuf ibn Matar (fl. 786–833) made his own Arabic translation in 827/828 based on a Syriac version of Sergios of Resaina (d. 536) (see Figure 1). Rose (1874: 333) consulted "… the beautiful Wallertstein manuscript …", where it states, with respect to the *Almagest*:

> This book was translated at the command of Maimon [al-Ma'mūn], king of the Arabs, who reigned in Baldalt [Baghdad], by Al Abhazez, son of Joseph, son of Matre the arithmetician [al-Hajjāj ibn Yusūf ibn Matar], and [previously] by Sergio, son of Albe the Christian, in the year 212 of the sect of the Saracens. (Our English translation.)[2]

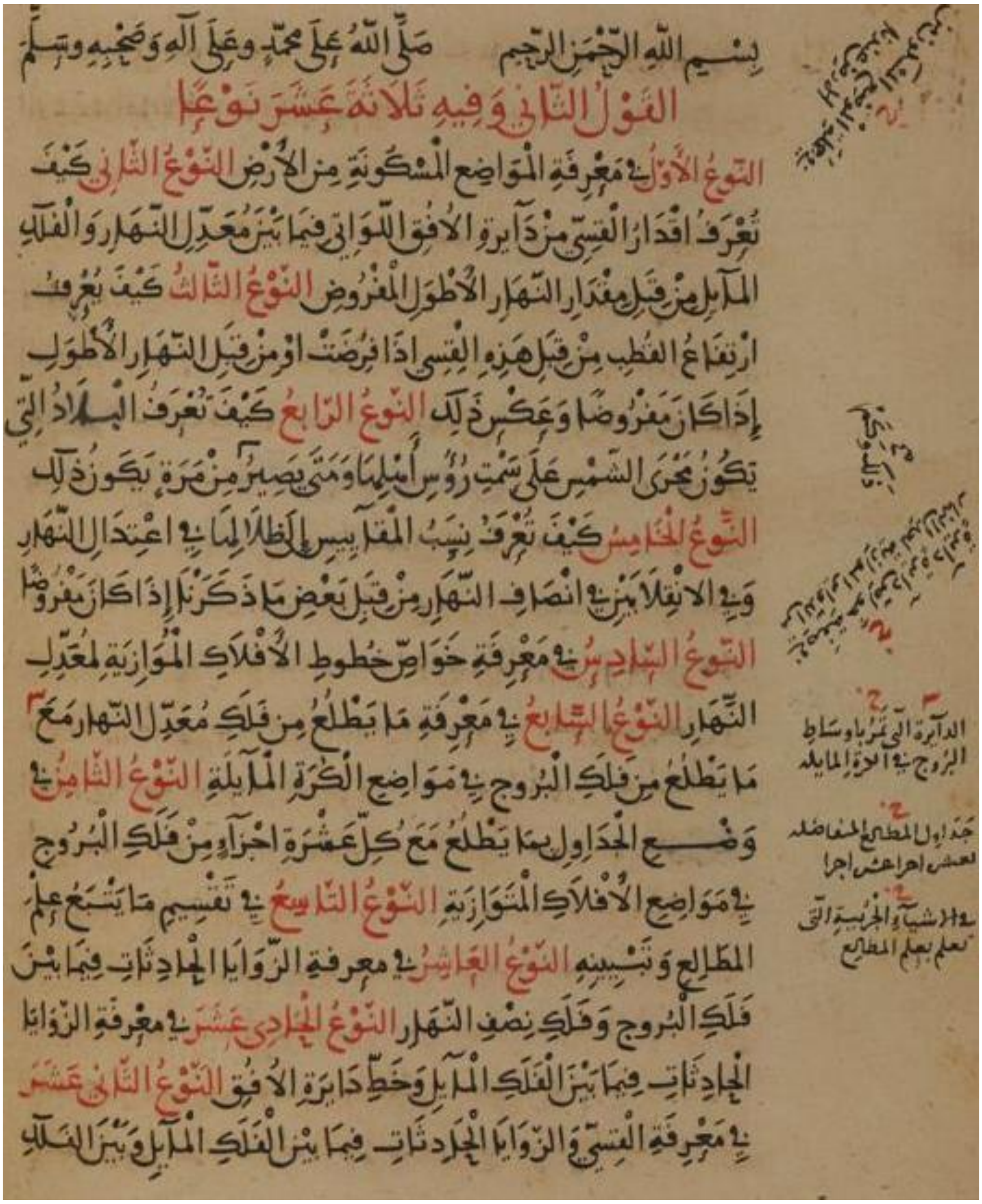
بسم الله الرحمن الرحيم صلى الله على محمد وعلى آله وصحبه وسلم
القول الثاني وفيه ثلاثة عشر نوعا
النوع الأول في معرفة المواضع المسكونة من الأرض النوع الثاني كيف
تعرف أقدار القسي من دائرة الأفق اللواتي فيما بين معدل النهار والفلك
المائل من قبل مقدار النهار الأطول المعروض النوع الثالث كيف يعرف
ارتفاع القطب من قبل هذه القسي إذا فرضت أو من قبل النهار الأطول
إذا كان معروضا وعكس ذلك النوع الرابع كيف تعرف البلاد التي
يكون مجرى الشمس على سمت رؤوس أهلها ومتى يصير من مرة يكون ذلك
النوع الخامس كيف تعرف نسب المقاييس إلى الأظلال لما في اعتدال النهار
وفي الانقلابين في انتصاف النهار من قبل بعض ما ذكرنا إذا كان معروضا
النوع السادس في معرفة خواص خطوط الأفلاك الموازية لمعدل
النهار النوع السابع في معرفة ما يطلع من فلك معدل النهار مع
ما يطلع من فلك البروج في مواضع الكرة المائلة النوع الثامن في
وضع الجداول بما يطلع مع كل عشرة أجزاء من فلك البروج
في مواضع الأفلاك المتوازية النوع التاسع في تقسيم ما يتبع علم
المطالع وتبيينه النوع العاشر في معرفة الزوايا الحادثات فيما بين
فلك البروج وفلك نصف النهار النوع الحادي عشر في معرفة الزوايا
الحادثات فيما بين الفلك المائل وخط دائرة الأفق النوع الثاني عشر
في معرفة القسي والزوايا الحادثات فيما بين الفلك المائل وبين

Figure 1: The beginning of Book II of al-Hajjāj ibn Yūsuf's translation of the *Almagest* (Ptolemy [827/828]).

The most widely used Arabic version of the *Almagest* was a translation by Ishāq ibn Hunayn (ca. 879–890), later revised by Thābit ibn Qurra (d. 901). The translations of al-Hajjāj and Ishāq-Thābit are still extant (Toomer, 1984: 2–3). According to Pedersen (1974: 15), the *Almagest*

> … gave rise to a large number of more or less revised versions, among which one of the most important was a long paraphrase by the Moorish astronomer Jābir ibn Afflah, named Geber by the Latins.

To quote Rose (1874: 333–334):

> In the manuscripts one usually finds afterwards the so-called Commentary of Geber ("Jābir, son of Aflah the Spaniard," according to the Nuremberg codex, see Boncompagni [monograph on Gerard of Cremona, 1851], p. 13) which Gerard had also translated in Toledo.

For his translation of the *Almagest* Gerard of Cremona (see below) relied on Books I–IX of the version by al-Hajjāj, and books X–XIII of the Ishāq-Thābit text. The star catalogue follows the form of the Ishāq-Thābit version.[3] For the most detailed available information on the Arabic translations of the *Almagest*, see Kunitzsch (1974: 115–125).

2 PTOLEMY FROM ARABIC AND GREEK TO LATIN

The earliest Latin translation of the *Almagest* was made in Sicily about 1160. Haskins and Lockwood (1910: 82–83) tell us:

> Of the name and nationality of the author of this translation nothing is revealed beyond the fact that he is a stranger to southern Italy and Sicily … Not only did the author of the Sicilian translation draw directly from the original Greek, but like other mediaeval translators from this language, he made a word-for-word rendering which, while not so painfully awkward and schoolboyish as the translations of [Henricus] Aristippus [d. 1162], is still very close and literal. For purposes of textual criticism a translation of this sort is not much inferior to a copy of the Greek text, and as there are but three existing manuscripts of the *Mathematike Syntaxis* anterior to the twelfth century, it is evident that our translation deserves careful collation and study.

Few copies of this translation were made, and its influence was minimal. One copy made in the fourteenth century or at the end of the thirteenth century (Vat. Lat. 2056) occupies 94 numbered folios. The prologue, amounting to 115 lines of Latin text, is reproduced by Haskins and Lockwood (1910: 99–102).

The notion that a formal school of translators functioned in medieval Toledo has been the subject of some debate (Bistué, 2013). It can be traced to Amable Jourdain's mention that a 'school of translators' was created in this city by Raymond of Toledo, who became Archbishop of this city ca. 1124 (Jourdain, 1843). Valentin Rose's article has a fundamental place in this narrative, since it offers the first search for evidence to support its existence (Rose, 1874). Rose's article is written in a style which has been described by native-German-speaking colleagues of ours as "very difficult" to "untranslatable", but because much of it is relevant to the transmission of Ptolemy's treatise to Renaissance Europe, we rely on it liberally here. To begin:

> Toledo was the place in which the threads of Platonic-Christian and Aristotelian-Arab science were woven together during a minimum of a century (ca. 1150-1250), and especially during the correspondingly long reign of Alfonso VIII (1158-1214). It was the seat of Christian power in Spain, having been conquered by Alfonso VI in 1085. It was the capital of the Castilian region and the most important place in [Christian] Spain. For all Europe it was the hotbed of the "doctrine of the Arabs." The colossal revolution of the era, the changed face of scientific activity, as if affected by a magic touch, [and] the fruitful zeal of the thirteenth century [shifted] to a previously unforeseen field of work, by which a new self-standing but comparatively humble spirit [of] the twelfth century [and its] applied offshoots evolved from [these] youthful foundations, such that everything was tied together in its origin in this city, in which, at the borders of the Arab world and on the old foundations of Arabic education, the whole Western world was attracted with wonder to the evidence of this. Here there were Arabic books in abundance as part of an [established] place of scientific scholarly activity [carried out by] a plethora of bilingual people. With their help Arabo-Christians (Mozarabs) and long-settled Jews developed here a formal school of Arabic to Latin book and scientific manuscript translation, whereby those people eager to know science sought to learn Arabic and to participate in the work. Numerous translations of the most famous writers of Arabic literature bear witness to this in Toledo. Englishmen and Germans, as well as Italians, linked the glory of their [careers] to their presence in this exalted school of Arabism and Arab science.
>
> The most productive of all translators who regularly brought new substance to the workmanship regarding all sciences, mathematics and astronomy, philosophy and medicine, Gerard of Cremona from Lombardy, spent almost his whole life in Toledo, learning and learning, translating and reading before disciples of the whole world, who pursued the same purpose more and more here. He was effectively the father of translators ("… who was the first among them …" said Roger Bacon). (Rose, 1874: 327–328.)

Other significant translators who worked in Toledo during 1150 to 1250 included Alvredus Anglicus (Alfred of England), Michael Scotus (Michael Scot), and Heremannus Alemannus the Bishop (Hermann the German):

> For example, in 1217 Michael Scot translated Alpetragius and then some time before 1230 translated, with the help of one Andreas ludaeus, Ar[istotle's] *De caelo et mundo* [On the Heavens] and other physical writings of Averroes [relating to] Aristotle. (Rose, 1874: 328–329.)

Hermann (ca. 1240) and Michael Scot both translated with the help of Arabs (Rose, 1874: 328).

The life of Gerard of Cremona (1114–1187) and the work in Toledo are described by Lemay (1978) and Burnett (2001). After Gerard's death his *socii* (colleagues and students) compiled a list of his translations and a brief account of his life, appending it to his last translation, the *Tegni* of Galen, with a commentary of Alī ibn Ridwān (Burnett, 2001: 254–256; Rose, 1874: 334). From the biographical account of Gerard we understand that it was the *Almagest* itself that drew him to Toledo: "For the love of the *Almagest,* which he could not find among the Latins, he made his way to Toledo." (our English translation; see Rose, 1874: 334 and Burnett, 2001: 255).

According to Lemay (1978: 174), Gerard arrived in Toledo at age 25 or 30, by the year 1144 at the latest. He was not familiar with Arabic at the time of his arrival. Thus, he had to work his way up to attempting his ultimate goal.

An Englishman named Daniel of Morley was an eyewitness to how Gerard eventually worked on the *Almagest*. He wrote a brief work entitled *Philosophia sive Liber de Naturis Inferiorum et Superiorum*, which was dedicated to Bishop John of Norwich. Since the Bishop (also known as John of Oxford) served from 1175 to 1200, this narrows down the time frame. Daniel's work comprises folios 88 to 103 of the Codex Arundel 377 in the British Museum,[4] and

> In the preface [Daniel] speaks of it that he had been out of England for a long time, studying in Paris, but, soon unsatisfied with the scanty erudition of the teachers there, went to Spain in order to hear the "wisest philosophers of the world" at Toledo, the famous center of Arab science … With a rich collection of books he would return to England, where the liberal arts lay in a deep slumber. (Rose, 1874: 330.)

According to Rose (1874: 334), Daniel was in Toledo between 1175 and 1187. On folio 103 of Daniel's work (Rose, 1874: 348) we find this very important phrasing: "… Gerard of Toledo rendered the *Almagest* into Latin with the Mozarab Galib as the one who was interpreting it."

The Mozarabs were Arabized non-Muslims, and Mozarabic was a Romance dialect spoken in Spanish territories under Arab domination. "The Mozarab translates naturally into Spanish, and Gerard into Latin according to this guidance …" (Rose, 1874: 335). Menocal (2002) remarks on the process of collaborative translation in Toledo:

> The common model was for a Jew to translate the Arabic text aloud into the shared Romance vernacular, Castilian, whereupon a Christian would take that oral version and write it out in Latin.

Lemay (1978: 174) notes that Galib is not named as a collaborator on any of Gerard's other translations. He writes: "… the assumption that translators usually worked in pairs is an undue extrapolation from the very scanty occurrences." Yet, it was Daniel of Morley's first-hand account of oral dictation that led Rose to assert that Gerard's translation of the *Almagest* was carried out "… in mündlichen Dictate …" (Rose, 1874: 335, note 3).

One final consideration was whether two translators worked together at the same time, or worked on the same manuscript at different times. It is believed that Daniel's use of the present participle form (*interpretante*) indicates that Gerard and Galib worked simultaneously (Bistué, 2013: 61).

We mention in passing that Roger Bacon did not have a high opinion of Gerard of Cremona, Hermann the German, Michael Scot, and Alfred of England. Apparently, he believed that the collaboration with translators who did not know Latin diminished the quality of the translation (Bistué, 2013: 63).

Rose (1874: 334, note 1) quotes the well-known statement given at the end of Gerard's translation of the *Almagest*: "Master Geradus of Cremona translated this book in Toledo from Arabic into Latin." (see Bonc[compagni, 1851]: 16 and 5) We do not know the exact date that it was finished, only an upper limit. A particular copy of Gerard's translation made by one Thaddeus of Hungary is dated 1175. This manuscript is kept at the Laurentian Library in Florence (Haskins and Lockwood, 1910: 78, note 1; Lemay, 1978: 174).

Many copies of Gerard's translation of the *Almagest* were made. For example, the Bibliothèque Nationale has ten copies (MSS. Lat. 7254-7260, 14738, 16200, 17864). MS. Lat. 14738 is from the close of the twelfth century (Haskins and Lockwood, 1910: 84, note 2). Gerard's translation was printed in 1515 in Venice by Petrus Liechtenstein. It, and the *Epitome of the Almagest* (see below), served as the foundation stones of astronomy upon which Copernicus built a new model of the Solar System.

Now, the reader would be correct to wonder why Renaissance astronomers would follow such a circuitous route to the astronomical knowledge of ancient Greece. Would it not have been better to make a definitive Greek to Latin translation, skipping Arabic and Mozarabic? Indeed, nearly 300 years after the Sicilian translation of 1160, this was attempted in 1451 by George of Trebizond (1395–1484). However, George's translation contained a considerable number of errors. Cardinal Johannes Bessarion (1403–1472), a famous Humanist of that era, expressed his antagonism toward George in a work called *In calumniatorem Platonis* (Pedersen, 1974: 20). Regiomontanus (see below) also wrote a polemic against George, which was never published, but the manuscript exists in St. Petersburg, Russia (Rosen, 1975: 351). Still, George's translation was considered significant enough to be printed by Giunti in Venice in 1528 (Pedersen, 1974: 21).

We turn now to the *Epitome of the Almagest,* written by Georg Peurbach (1423–1461) and his student Johannes Regiomontanus (1436–1476). See Hellman and Swerdlow (1978), Rosen (1975), and Shank (2017). Cardinal Bessarion wished to bring into existence a new Latin abridgment of the *Almagest*. At the start of their acquaintance Peurbach did not read Greek, but he knew Gerard's translation almost by heart (Hellman and Swerdlow, 1978: 474–475; Shank, 2017: 89). By the time of his premature death Peurbach was able to complete the first six books. The final seven books of the manuscript and a revision of the first half were finished by Regiomontanus in 1462 or 1463. It was published in Venice in 1496.

According to Shank (2017: 90):

> The *Epitome* is neither a translation (an oft-repeated error) nor a commentary but a detailed, sometimes updated, overview of the *Almagest*. Swerdlow once called it "the finest textbook of Ptolemaic astronomy ever written." It granted Bessarion's wish for a "condensed and clearer" exposition of Ptolemy, a constraint that explains why the work omits some of Regiomontanus's own cherished views …
>
> As the work of a practicing astronomer, however, the *Epitome* sometimes updates the *Almagest* by commenting on post-Ptolemaic developments. Regiomontanus discusses improved parameters and brings newer observations and theoretical work from the Islamic astronomical tradition to bear on his exposition, making extensive use of Albategnius (al-Battānī) and Geber (Jābir ibn Aflah) in particular.

Shank says that the *Epitome* is an 'overview'. We understand in general what that means, but, curiously, that word is not included in the lead author's abridged dictionary. In any case, the *Epitome* is not just a subset of quotations from the *Almagest*. It is a restating of many aspects of Ptolemaic astronomy, including ideas of medieval Arab astronomers.

3 COPERNICUS AND SOME CURIOSITIES

The founder of modern astronomy was the Polish astronomer Nicholas Copernicus (1473–1543). In 1491 he began his university studies in Cracow, where he attended lectures on the work of Albert of Brudzewo. He lived in Italy from 1496 to 1500, where he enrolled in the University of Bologna, officially to study canon law. In Bologna he was mentored in astronomy by Domenico Maria Novara, who had corresponded with Regiomontanus. Copernicus made a second trip to Italy from 1501 to 1503 to study medicine in Padua. He obtained a doctoral degree in canon law from the University of Ferrara in 1503, then returned to Poland where he spent the rest of his life (Goddu, 2008; Rosen, 1971).

What Copernicus learned in Cracow and Italy, along with the model set forth in Gerard's translation of the *Almagest* and the *Epitome,* formed the basis of his understanding of astronomy. Hellman and Swerdlow (1978: 477) note:

> The *Epitome* is the true discovery of ancient mathematical astronomy in the Renaissance because it gave astronomers an understanding of Ptolemy that they had not previously been able to achieve. *Copernicus used it constantly.* [Our italics.]

We do not know exactly how Copernicus came to believe the hypothesis that the planets and the Earth revolve around a stationary Sun, but he first laid out the basics of his geodynamic, heliocentric model in *De Hypothesibus Motuum Coelestium a se Constitutis Commentariolus* between 15 July 1502 and 1 May 1514 (Rosen, 1971: 406). It was not intended for publication, only circulation amongst a small number of trusted friends. A translation, with considerable commentary, is presented by Swerdlow (1973). Other editions have been presented by Edward Rosen (Copernicus, 1985) and Jerzy Dobrzycki (Copernici, 2007).

Copernicus owned a copy of the 1515 printed edition of Gerard's translation of the *Almagest*, and he also owned a copy of the first Greek-language edition of the *Almagest*, which was printed in Basel in 1538 (Gingerich, 2004: 16, 40–41). They are kept in Uppsala, Sweden.

Copernicus worked on his book *De Revolutionibus Orbius Coelestium* (*On the Revolutions of the Heavenly Spheres*) for the last 30 years of his life. It was published in an edition of 400 to 500 copies (Gingerich, 2004: 129) in 1543, the year of the author's death. It is one of the most significant works in the history of science, as it lays out the case for a reordering of the cosmos.

Early on, in Chapter 10 of Book One, Copernicus commits himself to a fundamental change of paradigm (Rosen, 1978: 32):

> At rest, however, in the middle of everything is the sun. For in this most beautiful temple, who would place this lamp in another or better position than that from which it can light up the whole thing at the same time? For, the sun is not inappropriately called by some people the lantern of the universe, its mind by others, and its ruler by still others … Thus indeed, as though seated on a royal throne, the sun governs the family of planets revolving around it.

Copernicus' new model retained one fundamental feature of ancient Greek astronomy—uniform circular motion. He knew that each planet exhibited a range of distance from the Sun, and for that reason he also retained eccentrics and epicycles. Retrograde motion, however, was not explained using epicycles. It came about naturally as a result of the (monotonically decreasing) speed of the planets along their orbits as a function of distance from the Sun, and the fact that we observe the universe from a planet that also orbits the Sun.

There is no substantial evidence that Copernicus owed the notion of a heliocentric system to anyone. Rosen (1975: 351), however, quotes this statement in the handwriting of Regiomontanus: "The motion of the stars must vary a tiny bit on account of the motion of the earth." This statement was excerpted by one Georg Hartmann (b. 1489), who recognized Regiomontanus' writing, but neither the letter nor the excerpt has survived, only a note written in the margin of an unpublished lecture of 1613. Rosen (1975: 352) concludes his brief biography of Regiomontanus as follows:

> Nevertheless, it has been suggested that the letter in question may have been sent by Regiomontanus to Novara, who, in an unpublished essay on the duration of pregnancy, called Regiomontanus his teacher. Novara in turn became the teacher of Copernicus. Thus it can be inferred that the concept of the revolutionary geokinetic doctrine was first conceived by Regiomontanus and communicated to Novara, who then passed it on to Copernicus. Nevertheless, in the voluminous published and unpublished writings of Regiomontanus, no other reference to the earth in motion has ever been found.

What the sentence attributed to Regiomontanus implies, of course, is the phenomenon of trigonometric stellar parallax. Astronomers of the Renaissance did not know that the nearest night-time stars are hundreds of thousands of Astronomical Units away, so their annual shifts of position are less than one arc second. It was not until the 1830's that astronomers were able to measure stellar positions to a small enough fraction of an arc second to demonstrate the parallax effect (Hirshfeld, 2013).

We can, however, puzzle over the possible influence on Copernicus by thirteenth and fourteenth century astronomers from the Middle East. This is curious because their documents were written in Arabic, a language Copernicus did not read. How could some of their work have come to the attention of Western European astronomers?

In 1259, following the establishment of the Ilkhanate, a part of the greater Mongol Empire, an observatory was established in its first capital, Marāgha, in the northern part of modern-day Iran (Sayili, 1960). The observatory was directed by the astronomer Nasīr al-Dīn al-Tūsī (1201–1274). This was the first astronomical institution worthy of the title 'research institute'. There they built and observed with a variety of instruments, copies of which (or blueprints) were transported to China (Hartner, 1950). Though the activity tailed off and eventually stopped after the death of Nasīr al-Dīn, Marāgha inspired the fifteenth century astronomer and prince Ulugh Beg (1394–1449) to build an observatory and carry out important work in Samarkand, in modern-day Uzbekistan (Krisciunas, 1988: 23–35; 1992).

The astronomers of the 'Marāgha School' objected to Ptolemy's use of the equant because it violated the notion of uniform circular motion in units of distance per unit time (Saliba, 2007: 95 ff.). To account for the details of the motion of the planets, they needed oscillating, linear motion. Such a thing is counter to the Aristotelian idea that linear motion is only possible in the sub-lunar realm. So how can one account for an oscillating linear offset, if it takes place in the celestial realm, by means of a combination of circles? Such a method was invented by Nasīr al-Dīn al-Tūsī. He showed how a smaller circle can turn inside another circle with twice the diameter, and a point on the diameter of the larger circle can oscillate back and forth linearly, even though it is produced through the combination of two uniform circular motions. This has been known as a 'Tūsī couple' since the 1960's (Kennedy, 1966). For an account of the use of the Tūsī couple up to the time of Copernicus, see Ragep (2017).

Another astronomer in Marāgha was Mu'ayyad al-Dīn al-`Urdi (d. 1266). He formulated a geometrical lemma that can be expressed as follows:

> Given any two equal lines that form equal angles with a base line, either internally or externally, the line joining the extremities of those two lines would be parallel to the base line. (Saliba, 2007: 152.)

`Urdi's Lemma, as it is known, was used (without proof) by Copernicus (Saliba, 2007: 154).

Ibn al-Shātir (1304–1375) of Damascus was an astronomer of the fourteenth century whose importance to historical discussions continues to grow, ever since Roberts (1957) showed that Ibn al-Shātir's theory of the Moon's motion is, for all intents and purposes, identical to that of Copernicus. Ragep (2016: 395) argues that Ibn al-Shātir's planetary models

> … in fact have a 'heliocentric bias' that made them particularly suitable as a basis for the heliocentric and 'quasi-homocentric' models found in [Copernicus's] *Commentariolus*.[5]

We are unqualified to referee the technical discussions on the planetary models of Ibn al-Shātir in the recent literature and refer the reader to the papers of Ragep (2016), Swerdlow (2017), and Nikfahm-Khubravan and Ragep (2019).

A non-technical opinion on this is expressed by Saliba (2007: 193):

> … the unintended consequences of these unified models produced a "strange" development that allowed them to be transferred into heliocentric models, despite the fact that there was no shred of support for such heliocentrism in the then reigning Aristotelian cosmology. All that someone like Copernicus had to do was to take any of Ibn al-Shātir's models, hold the sun fixed and then allow the Earth's sphere, together with all the other planetary spheres that were centered on it, to revolve around the sun instead … [That] was the very step that was taken by Copernicus when he seemed to have adopted the same geocentric models as those of Ibn al-Shātir and then translated them to heliocentric ones whenever the situation called for it.

Gingerich (pers. comm., 23 March 2017) comments:

> I vehemently disagree that the Ibn al-Shātir geometry would have inspired a heliocentric arrangement. On the other hand, I have lately become sorry that I didn't explain in my recent Copernicus biography [Gingerich, 2016] how it might have been very useful to Copernicus later on after he was exploring the heliocentric arrangement. If he just took the Ptolemaic arrangements and stacked all the planetary apparatus around the Sun, there would be the huge and unseemly clutter of all the equants near the Sun. Using the Ibn al-Shātir arrangements, each mechanism is associated closely with the individual planets. Copernicus could have worked this out for himself once given a hint. It may have saved the heliocentric transformation, but there is no good reason to think it might have engendered it. I can imagine Copernicus was very excited when he realized this move would tidy up his system, and he may never have heard of Ibn al-Shātir.

In 1957 Otto Neugebauer discovered a Greek-language manuscript in the Bodleian Library in Oxford that is relevant and very important, as it has stimulated discussion and historical research into the transmission of astronomical ideas from Marāgha/Tabriz/Asia Minor to Western Europe. According to Kennedy (1966: 378) it is a "… representation of a non-Ptolemaic device for determining the solar anomaly." Swerdlow (1973: 424, note 3) says that Neugebauer found

> … figures of a model using Tūsī's device for generating rectilinear motion from a circle rolling on the internal circumference of a circle of twice its radius.

The Greek-language document found by Neugebauer now resides in the Vatican (Vaticanus Graecus MS 211). Ragep (2014: 239) says that Vat. gr. 211 contains diagrams of the Tūsī couple and Tūsī's lunar model. Unfortunately, an internet search for images of these figures was not successful.

Vat. gr. 211 was written before 1308 (Paschos and Sotiroudis, 1998: 19) and may have been the work of the Byzantine astronomer Gregory Chioniades (ca. 1240–1320); see Neugebauer (1975: 11, 1035, 1456). Chioniades spent time in Tabriz (the second capital of the Ilkhanate), where he was mentored by Shams Pouchares (aka Bukharos); see Ragep (2014). This ties Chioniades to the geocentric model developed in Marāgha.

Hartner (1973) first drew attention to the remarkable similarity of a diagram from a work of Nasīr al-Dīn al-Tūsī from 1247 and a diagram found in Book 3, chapter 4 of *De revolutionibus* (Rosen 1978: 146). See Figure 2.

Given that the Arabic letter *alif* is the equivalent of A, *beh* is the equivalent of B, *daal* is D, *heh* is H, and *djim* is G (or J, if we use the modern alphabet), it is obvious to Hartner, to Saliba (2007: 199–201) and to us that Copernicus somehow became familiar with the diagram from Nasīr al-Dīn. However, not everyone agrees. Blåsjö (2014: 186) states that Hartner's claim of a similarity of the two diagrams is "… plainly an exaggeration, and a closer examination only undermines it further." We suspect that a statistical argument can be made that gives the probability that the two diagrams have nothing (or something) to do with each other, but this is beyond the scope of the present paper.

Further evidence of transmission of astronomy from the Middle East to Western Europe includes the finding that some of the al-Tūsī material is known to have reached Rome in the fifteenth century (Gingerich 1986: 83; Guessoum, 2008). There is no evidence that Copernicus saw it.

Langermann (2007) and Morrison (2014) have written on the Byzantine Jewish scholar Moses Galeano, who spent time in the Veneto be-

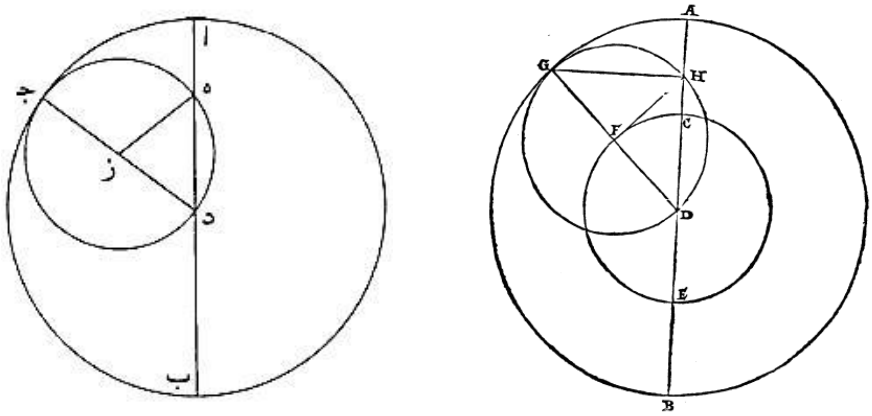

Figure 2 (Left): Diagram from the *Tadhkira* (1247) of Nasīr al-Dīn al-Tūsī (after Ragep, 1993: 199). Right: Analogous diagram from Copernicus' *De Revolutionibus* (1543) (after Hartner, 1973).

tween 1497 and 1502. This is

> … the first piece of evidence that someone who read Ibn al-Shātir and was quite conversant in mathematical astronomy was present in Italy at the turn of the sixteenth century. (Langerman, 2007: 290.)

This intriguingly overlaps the time that Copernicus himself was in northern Italy. We have no hard evidence that Galeano met Copernicus or his teacher Novara, but we may hypothesize that there was direct or indirect contact. How might this have come about? After the end of the Byzantine Empire in 1453, Christians were less welcome in Constantinople. Many left, and scholars would have brought books and manuscripts with them. We know very little about who they were and what they did. Ragep (2017: 194) points out that the Ottoman sultan Bāyazīd II was a patron of Christian, Jewish, and Muslim scholars. Galeano worked for the Ottoman sultan for a good part of his career (Langerman, 2007: 286). He had the freedom to come and go. Meanwhile, there was a long-standing presence of Jews at the University of Padua and a significant Jewish community there (Morrison, 2014: 36).

Five centuries after Copernicus it may be impossible to determine how some ideas found their way into *De revolutionibus.* Dobrzycki and Kremer (1996: 211) suggest:

> We may be looking for a means of transmission both more fragmentary and widespread than a single treatise, and at least one of the Marāgha sources must have been available to the Latin West before 1461, the year of Peurbach's death.

Guessoum (2008: 235) points out that in *De Revolutionbus* Copernicus cites al-Battani, al-Bitruji, al-Zarqali, Ibn Rushd (Averroes), and Thābit ibn Qurra, but does not mention Nasīr al-Dīn al-Tūsī or other Marāgha astronomers. Guessoum (ibid.) writes:

> Perhaps the answer is that some of the Marāgha material that [Copernicus] used reached him without proper reference and/or *via* medieval western sources.

Goddu (2018: 201) believes we should consider a category between direct copying or independent development. He calls it 'idea diffusion'.

4 DISCUSSION AND CONCLUDING REMARKS

In this paper we have endeavored to trace the transmission of Ptolemy's *Almagest* up to the time of Copernicus. As history unfolded, this involved the translation of many manuscripts from Greek to Arabic in ninth century Baghdad, and, just as significantly, it involved the committed work of translators in Toledo from ca. AD 1150 to 1250. Thus,

> There was no need to translate anything on grammar or rhetoric, theoretical arithmetic or music, because the Latins were well supplied with texbooks on these subjects. The main gaps were the remaining parts of rhetoric and dialectic, geometry and astronomy … What remains to be explained is the driving force behind this translation enterprise [in Toledo] … What is beyond doubt is the scale and importance of the enterprise, which has no match in the history of western culture. (Burnett, 2001: 257, 269–270.)

Over a 43 year period in Toledo Gerard of Cremona translated over 70 works from Arabic into Latin. His translation of the *Almagest* into Latin led to the *Epitome of the Almagest* by Peurbach and Regiomontanus, which Copernicus frequently consulted. An historically inaccurate account of Copernicus's great accomplishment, the geodynamic, heliocentric model of the solar system, implies that after he studied the model of the ancient Greeks, he had a bold new idea, elaborated the details, and the astronomers of the sixteenth century said, "Why didn't we think of that?" But it was self-evident to most people that the Earth was stationary. Aristotle's physics stipulated that the Earth was the center of the cosmos. The medieval Arab astronomers had no interest in a moving Earth model. Copernicus changed that.

Could Copernicus have thought of the radical new model entirely on his own? Yes. As Blåsjö (2014) implies, we accept that Kepler worked out his three laws of planetary motion on his own, and Newton worked out the Law of Gravity and showed that Kepler's laws derive from the gravitational force law. What intrigues us, and many others, is that Copernicus's *De Revolutionibus* appears to make use of some details of the models from thirteenth century Marāgha and fourteenth century Damascus. If we can understand how that happened, we will achieve a better focus on the birth of modern astronomy.

Respected researchers in this field have laid out the case of some influences on Copernicus. In his commentary on Copernicus's *Commentariolus*, in a section on the motion of Saturn, Jupiter, and Mars, Swerdlow (1973: 469) writes:

> One may seriously wonder whether [Copernicus] understood the fundamental properties of his model for the first anomaly, and this of course bears strongly on the important question of whether the model was his own invention or something he learned of from a still undiscovered transmission to the west of a description of Ibn [al]-Shātir's planetary theory. My own inclination is to suspect the latter, not because I think Copernicus incapable of carrying out such an analysis of the first anomaly in Ptolemy's model (he certainly shows considerable ingenuity in deriving the heliocentric representation of the second anomaly), but rather because of the identity with the earlier planetary theory of Copernicus's models for the moon *and* the variation of the radius of Mercury's orbit *and* the generation of rectilinear motion by two circular motions seems *too remarkable a series of coincidences to admit the possibility of independent discovery.* [Our italics.]

Saliba (2007: 209) further quotes Swerdlow on the subject of Mercury's motion:

> The transmission of their [meaning the Marāgha astronomers] inventions from Arabic in the East to Latin in the West is obscure. Yet Copernicus's lunar and planetary theory in longitude in the *Commentariolus*, right down to the additional complications for Mercury, is that of Ibn al-Shātīr in nearly every detail, except for the heliocentric arrangement and the extraction of parameters from the *Alphonsine Tables*, and it is hard to believe in light of so many and such complex identities that Copernicus was entirely without knowledge of his predecessors.

Blåsjö (2014) rejects essentially every influence on Copernicus suggested over the past 60 years. See Ragep (2017: 184–193) for a review of the evidence for and against transmission from the Mideast to Western Europe. Given the similarities of the lunar and planetary models of Nasīr al-Dīn and Copernicus, the use of `Urdi's Lemma by Copernicus, and the argument (above) relating to the similarity of the labeling in a diagram found in Nasīr al-Dīn's *Tadhkira* of 1247 and a diagram in *De Revolutionibus*, the case that Copernicus was familiar with some of the aspects of Middle Eastern astronomy of the thirteenth and fourteenth centuries is, in our view, very strong.

Ragep (2017: 184) writes:

> Although difficult to gauge in a precise way, impressionistically it seems that a majority of historians of early astronomy have accepted, to a lesser or greater degree, the influence of late-Islamic astronomy on early modern astronomers, particularly Copernicus. This acceptance is perhaps most explicitly set forth by Swerdlow and Neugebauer: "The question there is not whether, but when, where, and in what form he [Copernicus] learned of Marāgha theory."

Morrison (2017: 213–214) concludes:

> The translation of tables and [the] numerous contacts between Jewish astronomers and Christian (and Muslim) astronomers in Renaissance Europe, the Byzantine Empire, and the Ottoman Empire mean that *contact between astronomers on matters of theoretical astronomy is more plausible than a presumption of no contact* … The work of Renaissance astronomers, including Copernicus, should be understood as a continuation of astronomy in Jewish and Islamic civilization (and in late-medieval Europe), not as a radical disjuncture with the past. [Our italics.]

5 NOTES

1. The Sun was treated differently, using an *eccentric* (a circle whose center does not coincide with the Earth).
2. The year 212 of the Muslim calendar ran from 2 April 827 through 20 March 828 of the Christian calendar (see https://habibur.com/hijri/ accessed 4 August 2019). The authors of this paper were responsible for Latin to English translations of all Rose (1874) quotations given in this paper.
3. Kunitsch, P., n.d. https://www.qdl.qa/en/arabic-translations-ptolemys-almagest (accessed 7 August 2019). The online Qatar Digital Library, maintained by the Qatar National Library in partnership with the British Library, is an excellent source of more information on Arabic manuscripts, but beyond the scope of the present article.
4. *https://www.bl.uk/catalogues/illuminatedmanuscripts/record.asp?MSID=1773&CollID=20&NStart=377* (accessed 5 August 2019).
5. What Copernicus meant by 'homocentric' is a common, fixed, center that is itself eccentric.

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